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ABSTRACTS

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A Domain Decomposition Method for Singularly Perturbed Boundary Value Problems with a Local Distribution of the Initial Conditions

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We present a numerical method for a Dirichlet problem for a singularly perturbed parabolic equation with a local perturbation of the initial condition. The differential equation, in which the highest derivative is multiplied by a small parameter ϵ, is defined on an interval. The initial condition of the problem has a local perturbation of finite amplitude over a small area of width 2δ.

When ϵ vanishes, the differential equation degenerates, and only the time derivative remains. For small values of the parameter boundary layers appear, and for $\delta < \epsilon$ also interior layers. In this case the derivatives of the solution increase without bound.

For small values of the parameters ϵ and δ, difficulties appear when we want to solve the problem numerically. It is well known that the classical difference schemes do not converge ϵ-uniformly, even for $\delta = 1$. It is shown that, also for $\epsilon = 1$, the classical difference scheme does not converge uniformly with respect to the small parameter δ (δ-uniformly).

A special finite difference scheme that does converge, ϵ-uniformly and δ-uniformly, is constructed. For this it is necessary to use grids that concentrate in the neighbourhood of the local perturbation of the initial data. Based on this local grid refinement domain decomposition is applied, and an alternating Schwarz method is used to solve the boundary value problem. With numerical examples it is shown how the small parameters ϵ and δ, and the computational parameter in the Schwarz process influence the numerical solution.